Optimisation Strategies for Decentralized ASAC

Leopoldo P.R. de Oliveira, Bert Stallaert, Wim Desmet, Jan Swevers, Paul Sas
K.U.Leuven, Dept. of Mechanical Engineering, division PMA
Celestijnenlaan 300, B-3001 Leuven, Belgium
e-mail: **Leopoldo.deOliveira@mech.kuleuven.be**

In decentralised velocity feedback control of structures, the configuration of collocated sensor/actuator pairs has a large influence on the attainable reduction in vibration and sound radiation. In the field of active structural acoustic control, the understanding of the physical phenomena involved in the fluid-structure interaction is important. This paper presents a comparison of optimisation techniques for the placement of multiple sensor/actuator pairs. The goal of the controller is to achieve the maximum reduction of kinetic energy or total radiated sound power. The techniques are applied on a rectangular simply supported plate with varying numbers of sensor/actuator pairs.

1 Introduction

The main objective of active structural acoustic control (ASAC) is to actively influence the vibration of a structure in order to reduce the sound radiation [8]. However, the successful practical application of active control depends on many factors such as the physical objectives, the control strategy and even the nature of the disturbance [3].

In ASAC, not always the best result for sound radiation is obtained by reducing the vibration, but by changing the velocity patterns on the surface, in such a way that it results in a reduction of the total radiated sound. The controller efficiency depends directly on the adequate choice of the input and output parameters such as sensor and actuator location and the control strategy [3, 4].

This paper presents comparisons of different approaches for the placement of single and multiple collocated sensor/actuator pairs (SAPs) for direct velocity feedback controllers. The controllers are applied to a simply supported aluminium plate (247 x 278 mm). The disturbance is random in time and space so that all the mode shapes in the frequency range are excited with an uncorrelated random force [6].

Section 2 discusses the control strategy and the different cost functions that have been investigated. Two cost functions are presented; kinetic energy and total sound power.

In section 3, three approaches for the placement of a single SAP are discussed: an extensive search, a genetic algorithm (GA) and a direct approach, all of them followed by a feedback gain optimisation. In the extensive search, the optimal gain is calculated for every possible SAP location on a pre-defined mesh. The GA is a direct search method based on Darwin's "survival of the fittest" theories [3], resulting in a reduced computation time. The direct approach calculates a measure for the controllability and observability in each point on a pre-defined mesh.

Section 4 deals with the placement of multiple SAPs. Depending on the number of actuators, different placement strategies are possible. For two SAP's, the extensive search is compared with the GA. For four SAP's, the extensive search becomes too computationally expensive. Instead a GA is compared with an incremental optimisation and an optimal grid. The incremental optimisation consists of a sequence of optimisation steps. The optimal grid is a symmetrical placement of four SAPs with the same optimised feedback gain in each quadrant of the plate. In the end, results for multiple SAPs are compared with randomly placed ones.

2 Collocated velocity feedback

When no reference signal is available, it is necessary to use feedback control to drive measured error signals to zero. In a feedback configuration however, stability is not always guaranteed.

However, when the SAP is collocated and velocity feedback is used, it can be shown that the system is passive and stability is always guaranteed, independent of the feedback gain [8, 9, 10]. A collocated SAP means the distance between actuator and sensor is very small compared to the wavelengths of the modes to be controlled. The velocity feedback results in a passive system because the velocity sensor and force actuator are dual.

An essential element in the design of a controller is the choice of the cost function, which is used to tune the degrees of freedom of the control configuration; feedback gain and location of the SAPs. After defining the position of the SAPs, optimisation of the feedback gain is necessary to reduce the cost function to the greatest possible extent [7, 10]. The next sections show the objective functions for kinetic energy and total sound power based on modal amplitudes.

2.1 Kinetic energy

Assuming that the structural modal amplitudes are available in the form of the complex column vector $\mathbf{a}$, the expression for the plate's kinetic energy E_k can be written as [1]:

$$E_k = \frac{1}{8} m \, \mathbf{a}^H \mathbf{a} \tag{1}$$

where m is the plate total mass and superscript H denotes the conjugate transpose.

2.2 Total sound power

Methods for active control of sound radiated by vibrating structures usually fall in two categories: reduction of the acoustic pressure in a specified point or region, and reduction of the total sound power

As in [5], the total sound power W radiated by a structure can be written in terms of the real part $\mathbf{R}$ of the specific-acoustic impedance matrix or in terms of the modal amplitudes $\mathbf{a}$ and the radiation efficiency matrix $\mathbf{M}$:

$$W = \mathbf{v}^H \mathbf{R} \mathbf{v} = \mathbf{a}^H \mathbf{\Phi}^H \mathbf{R} \mathbf{\Phi} \mathbf{a} = \mathbf{a}^H \mathbf{M} \mathbf{a} \tag{2}$$

where $\mathbf{v}$ is the velocity distribution on the plate and $\mathbf{\Phi}$ is the mode shape matrix. $\mathbf{M}$ is a real, symmetric, positive definite matrix in which the diagonal terms are proportional to the self-radiation resistances and the off-diagonal terms are proportional to the mutual-radiation resistances:

$$\mathbf{M} = \mathbf{\Phi}^H \mathbf{R} \mathbf{\Phi} = \left(\frac{\rho c S_T}{2} \right) \begin{bmatrix} \sigma_{11} & \sigma_{12} & \dots & \sigma_{1m} \\ \sigma_{12} & & & \\ \vdots & & \ddots & \\ \sigma_{1m} & & & \sigma_{mm} \end{bmatrix} \tag{3}$$

where ρ is the air density, c the speed of sound and S_T the plate area.

Taking a closer look at the radiation matrix, it can be seen that above a certain frequency it becomes diagonal (see Figure 1) i.e., only the self radiation terms play a role in the total radiated sound power.

3 Placement of a single SAP

In this section, the control system consists of one collocated sensor/actuator pair, resulting in a single-input single-output system (SISO). Three approaches are compared to find an optimal location for the SAP: (i) extensive search, (ii) genetic algorithm and (iii) direct approach, all of them followed by a feedback gain optimisation.

The extensive search calculates the optimal gain, leading to the largest reduction in kinetic energy or radiated sound power, for every point in a mesh in one quarter of the plate (since it is symmetric). The GA also uses the kinetic energy or radiated sound power as cost functions to evaluate the fitness of each population after a gain optimisation, which is discussed in more detail in following subsection. Finally, the two last subsections present in more detail the direct approach for both kinetic energy and total sound power control.

3.1 Genetic algorithm

The placement of a single SAP is investigated with a genetic algorithm. Genetic algorithms belong to the so called direct random search techniques [3]. In this work the feasible positions for the SAP are restricted to points on a 31x31 mesh. Thus, one gene consists of sequences of bits encoding the x-position x_i and the y-position y_i for each SAP. For each position, optimisation of the feedback gain is necessary. Since this optimisation appeared to be convex, it was not included in the gene. Instead feedback optimisation was performed in each iteration of the GA, when new locations for SAPs were chosen. Equation 4 illustrates a gene in the case for one SAP where n depends on the number of points on the mesh in the x direction and m in the y direction.

$$\text{gene} = \begin{bmatrix} x_{1,1} & x_{1,2} & \dots & x_{1,n} & y_{1,1} & y_{1,2} & \dots & y_{1,m} \end{bmatrix} \tag{4}$$

For the estimation of multiple SAPs, the sequence is just repeated as many times as the number of SAPs.

The objective functions for the GA are kinetic energy and total sound power.

3.2 Direct approach for kinetic energy control

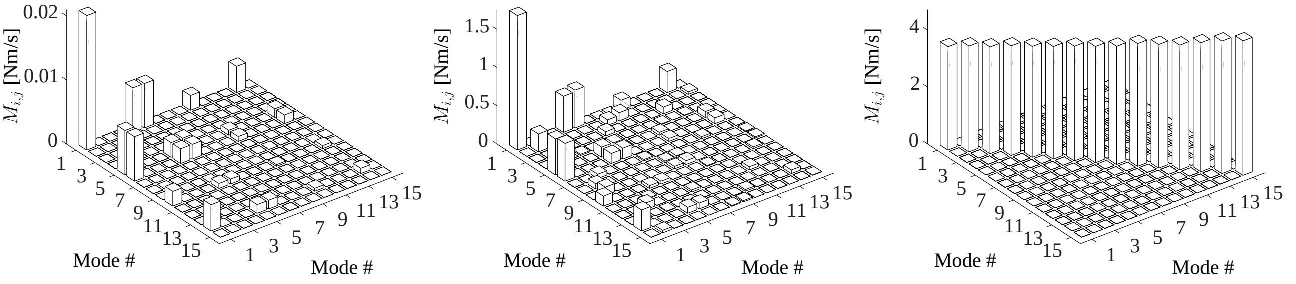
When performing a modal test, the proper placement of sensors and actuators is a key to appropriately excite all the modes of interest in a certain frequency range. This means that the exciter and sensors must be placed in points with good modal participation of all the modes of interest. From the control point of view it is similar to say that such points have good controllability and observability.

One of the objective functions used to obtain parameters that can describe this average modal participation is the average driving point degree of freedom (ADDoFV) [2].

The receptance H_{pk}, given by the displacement at p due to a point force at k can be defined as:

$$H_{pk}(\omega) = \sum_{r=1}^{M} \frac{\phi_{pr}\phi_{kr}}{\omega^2 - \omega_r^2 + j2\zeta_r \omega \omega_r} \tag{5}$$

where M is the number of modes of interest, ϕ_{pr} and ϕ_{kr} are respectively the value of the mode shape r at points



(a) 50 Hz (b) 500 Hz (c) 5000 Hz

Figure 1: Bar plots of $\mathbf{M}$ for 15 modes at various frequencies.

p and k, ω_r is the natural frequency and ζ_r the modal damping associated with mode r. After some manipulations, the velocity modal amplitudes $\mathbf{a}$ of a unitary force at point k in the vicinities of the mode r can be written as:

$$a_{kr} = \frac{\phi_{kr}}{2\zeta_r \omega_r} \tag{6}$$

The ADDoFV takes the average value of a for the modes on the frequency band of interest, and this in every degree of freedom:

$$\mathrm{ADDoFV}(k) = \frac{1}{N}\sum_{r=1}^{N} a_{kr} = \frac{1}{N}\sum_{r=1}^{N} \frac{\phi_{kr}}{2\zeta_r \omega_r} \tag{7}$$

A point with a large ADDoFV is a point where the controllability is large. Based on the reciprocity principle, the same can be said about the output velocity. Thus, from the control point of view, the ADDoFV indicates the best point for average controllability and observability. In this way, the ADDoFV gives a plot with the best average modal participation factors, and since it is a velocity average, the results will be closely related to the plate kinetic energy.

Figure 2 shows a comparison between the ADDoFV and the achievable reduction for a SAP in that point, with optimised feedback gain. The point with the largest reduction in kinetic energy obtained by the extensive search is the same as the point with the largest ADDoFV. This concludes that for one SAP, the same result is achieved by calculating the ADDoFV and optimising the gain than by doing an extensive optimisation, which is much more computationally expensive.

3.3 Modification of the direct approach for sound power control

Equation 2 shows the total sound power in terms of the squared modal amplitudes and the radiation efficiencies. Since the ADDoFV is based on an average of mode shapes, the modified version proposed here takes into account the influence of each mode in the total sound power as a weighting factor in the modal average.

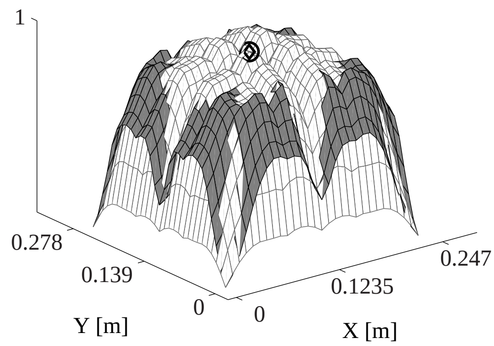


Figure 2: Comparison of maximum achievable energy reduction (gray) and ADDoFV (white); best place for ADDoFV (◇) and best place for extensive search (◯).

Equation 8 shows the weighting factors as a function of the modal amplitudes and the self-radiation efficiencies. In this way, the weighting factor is proportional to the influence of each mode in the radiated sound power near its natural frequency:

$$\Lambda_r = \left|\mathbf{a}_r(\omega_r)\right|^2 \sigma_{rr}(\omega_r) \tag{8}$$

where $\mathbf{a}_r$ is the modal amplitude vector for mode r.

Figure 3 shows the weighting factors for the first 17 modes, where it can be seen that odd-odd modes such as the 1^{st}, 5^{th} and 6^{th}, have higher influence than even-even such as the 4^{th}.

The result of the modified ADDoFV can be seen in Figure 4, where again it points to the same coordinates as the extensive search.

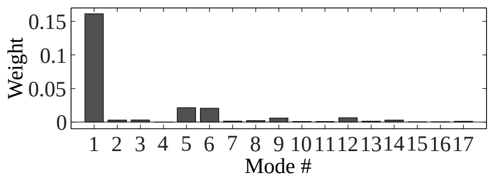


Figure 3: Weighting functions according to equation 8.

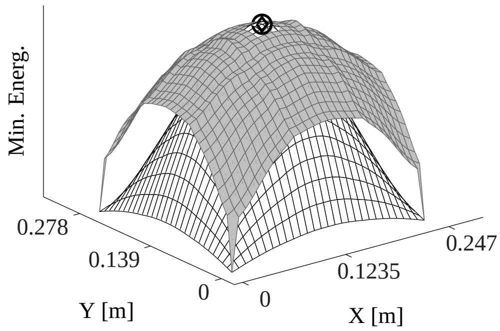


Figure 4: Comparison of maximum achievable total sound power reduction (gray) and ADDoFV (white); best place for ADDoFV (◇) and best place for extensive search (◯).

4 Placement of multiple SAPs

When placing multiple SAPs, the control system becomes a multiple-input multiple-output system (MIMO). However, in this paper only decentralised systems are considered, in other words, each SAP has only one feedback gain associated resulting in a diagonal feedback matrix. Controllers with two and four SAPs were investigated.

For two SAPs the extensive search is still feasible and it consists of the calculation of every possible combination of two SAPs locations with the optimisation of feedback gains. The GA for multiple SAPs is an extension of the one developed for a single SAP. As mentioned before, in this case, the sequence of bits in a gene is repeated as many times as the number of SAPs as in Equation 4.

For four SAPs, the extensive search was too computationally expensive. Therefore an incremental optimisation is performed, consisting of a sequence of optimisation steps, where in the n'th step, the system with $n-1$ SAPs already placed is considered as a new system. These results are compared with a GA and a uniform grid. The GA is supposed to give the global optimum. The uniform grid consists of the placement of n^2 SAPs, four in this case, equally spaced from the borders and from each other and with the same optimised gain.

To assess the necessity of optimisation, the optimised results are compared with the result obtained when placing the four SAPs randomly. The objective of this trial was to get an average value of the control effectiveness for a random choice of SAPs locations, in order to evaluate the effort of performing an optimisation. The only constraint was that each SAP should be placed in a different quadrant of the plate. After one hundred trials, the reduction mean value and standard deviation were computed.

5 Results

This section presents the results for a single and multiple SAPs. The disturbance is considered to be random in time and space, as a result, every mode is excited with the same amplitude.

5.1 Single SAP

The results for a single SAP are quite consistent over the different strategies. All three resulted in the same position and slightly different gains: 31.9 Ns/m for the GA and 31.3 for the others (Table 1). Figure 5 shows the open and closed-loop kinetic energy for the three cases.

Table 1: Results for 1 SAP – Kinetic energy

X [mm]	**Y [mm]**	**Gain [Ns/m]**	**Red. [dB]**
98.8	111.2	31.3, 31.9	4.7

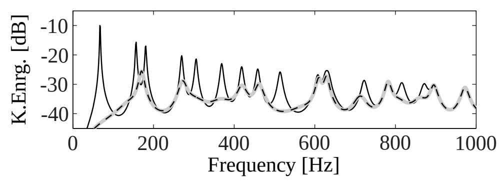


Figure 5: Comparison of kinetic energy for open-loop (solid-black), optimal gain (solid-grey) and GA gain (broken-black).

The results for total sound power cost function are presented in Table 2. Figure 6 shows the total sound power for these configurations.

Table 2: Results for 1 SAP – Total sound power

X [mm]	**Y [mm]**	**Gain [Ns/m]**	**Red. [dB]**
123.5	139.0	24.2	11.8

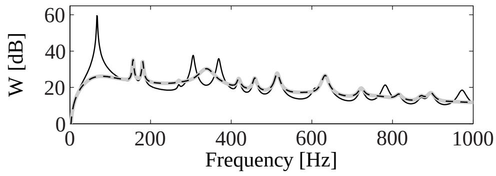


Figure 6: Comparison of total sound power for open-loop (solid-black), optimum gain (solid-grey) and GA gain (broken-black).

Observing the results for kinetic energy and total sound power, it can be seen that both cost functions lead to different controllers. A controller for kinetic energy tries to attenuate all resonances, while a controller for sound power focuses on those modes that radiate most efficiently and neglects modes that play a minor role in sound radiation (like the 2^{nd}, 3^{rd} and 4^{th}). This confirms that the greatest sound reduction is not obtained by a simple reduction of vibration levels.

5.2 Multiple SAPs

Figure 7 and Table 3 show the positions and feedback gains for the simultaneous placement of 2 SAPs according to an extensive optimisation and the GA with respect to kinetic energy and total sound power.

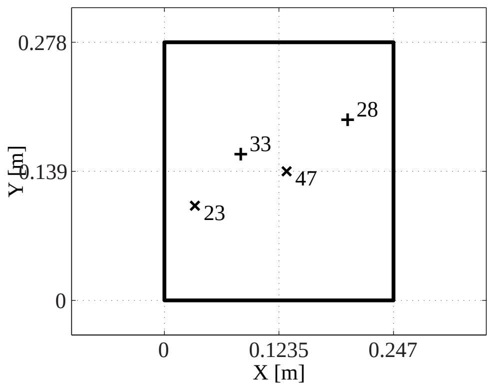


Figure 7: Plate top view with 2 SAPs and gains for optimal kinetic energy (+) and for optimal total sound power (x)

Table 3: Results for 2 SAPs

Method	SAP	X [mm]	Y [mm]	Gain [Ns/m]	Red. [dB]
E_k	1	82.3	157.5	32.2	7.3
	2	197.6	194.6	28.6	
W	1	32.9	101.9	22.9	12.5
	2	131.7	139.0	47.4	

Figures 8 and 9 show the results for optimal kinetic energy and sound power for 2 SAPs.

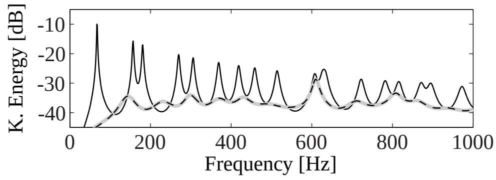


Figure 8: Comparison of optimal kinetic energy with 2SAPs for open-loop (solid-black), optimisation (solid-grey) and GA (broken-black).

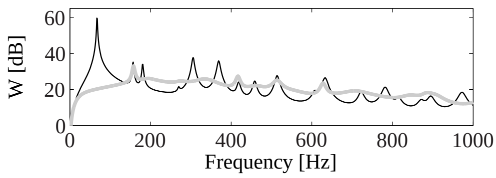


Figure 9: Comparison of optimal sound power for open-loop (solid-black) and GA (solid grey).

In addition, the results for the placement of 4 SAPs are shown in Figure 10. In this case the GA performs the simultaneous placement of the 4 SAPs. The optimal grid consists of 4 SAPs equally spaced and with the same gain and the incremental optimisation is the stepped placement of 4 SAPs. The last two strategies should point to sub-optimum solutions.

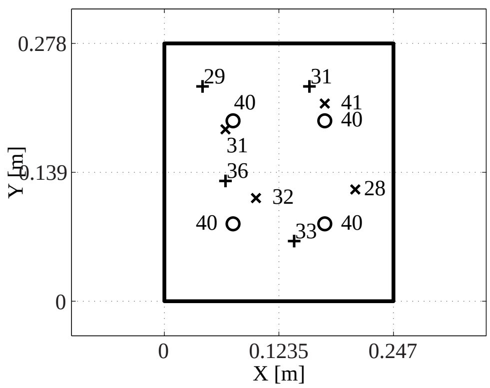


Figure 10: Plate top view with 4 SAPs and feedback gains for kinetic energy from GA (+), incremental optimisation (x) and the optimal grid (0).

Figure 11 shows a comparison between the resultant kinetic energy for these 3 cases. It can be seen that the incremental optimisation and the GA show similar behaviour; they damp all the resonance frequencies in the frequency band. In the grid configuration, the SAPs are located in the nodes of some of the modes and as a result these modes are not controlled.

Finally, the random placement of 4 SAPs resulted in an

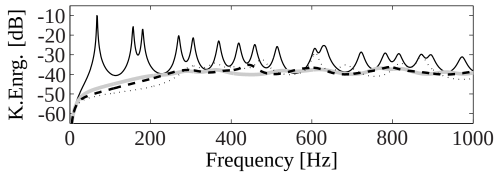


Figure 11: Comparison of kinetic energy for open-loop (solid-black), GA (solid-grey), incremental optimisation (broken-black) and grid (dot).

average 8.8 dB reduction in kinetic energy with a .38 standard deviation for 100 cases.

Table 4 shows the achievable reduction on the kinetic energy for the 4 cases, where the grid is the least efficient, the incremental optimisation and the GA show almost the same result and the random placement average is in between.

Table 4: Reduction in kinetic energy

Method	**Grid**	**Random**	**Incr. Opt.**	**GA**
Red. [dB]	8.1	8.8	9.8	10.0

6 Conclusions

Various strategies for the placement and optimisation of SAPs were compared.

For 1 SAP, it was shown that the proposed direct approach pointed to the global optimum. However it still has to be tried with more complex geometries and different disturbances to prove robustness.

In the case of 2 SAPs, the results show an improvement of 2.6 dB in kinetic energy and 0.7 dB in total sound power with respect to 1 SAP.

For 4 SAPs, the incremental optimisation and the GA showed almost the same reduction, indicating that this study case may present various local optima with similar performances which difficult the assessment of the global optimum. For this case the grid is the least efficient, even considering the random placement, which according to Table 4 presents an improvement of 0.7 dB in average.

According to this result for a quite simple structure, with idealised sensors, actuators and boundary condition, the benefits of using computational expensive approaches such as the GA can be questioned when compared to some sub-optimal solutions.

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