

# Optimisation of digitally adjustable analogue biquad filters in feedback active control

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In active cancellation of sound radiation from structures, feedback control is applied if no suitable reference is available. In feedback active control, analogue filters are typically used due to their short latency time. The filters have traditionally been simple compensators with fixed coefficients. Such compensators can be implemented as second-order biquad filters. This paper concerns the use of biquad filters with digitally adjustable parameters as analogue feedback compensators. The biquad filters can be implemented using a Field Programmable Analogue Array (FPAA) circuit. The benefit of the FPAA implementation is that the parameters are set digitally, and the zeros and poles as well as the corresponding quality factors can be easily modified. An optimisation method based on random search is proposed for optimising the coefficients of the biquad blocks. The optimisation objective is to minimise the magnitude of the transfer function from the disturbance to the error while the maximum amplification of the error is a constraint. For the design of the compensator, only the complex frequency response of the plant is required. The performance of the optimiser is evaluated with an optimiser provided in MATLAB. Using both optimisers, an optimal compensator is designed for a test plant. In this paper, the random search optimiser is described and the optimisation results are shown.

## 1 Introduction

Feedback active control is applied in cases where no reference signal providing a priori information about the disturbance is available. In active cancellation of sound radiation from structures, feedback control is employed when the excitation is neither deterministic nor can be detected at some time well before it excites the structure [1]. Analogue filters are typically used in feedback active control due to their shorter delay compared to the digital counterparts. The filters are usually simple first- or second-order compensators. Traditionally, the coefficients of the compensators have been fixed but with the current technology including digital potentiometers or Field Programmable Analogue Arrays (FPAA), they can also be made adjustable. An FPAA-based compensator has been implemented in a feedback active noise control system [2].

The model of the plant from the actuator to the error sensor plays an essential role in the design of a feedback compensator [3]. Accurate model of the plant is required to achieve good control performance. In acoustical system, however, accurate analytical models are difficult to obtain due to the complexity of the dynamic functions, propagation delays and non-minimum phase behaviour. A practical way to obtain a model of the plant is to measure the response from the actuator to the error sensor. The measured complex frequency response can be used as a frequency domain model of the plant. Identification techniques can be applied for estimating the model from time domain data as well.

For optimisation of the compensator, several methods have been developed. In standard optimal feedback

techniques, the linear quadratic Gaussian (LQG) approach has been applied for state-space models [1]. Carne proposed a method in which the sensitivity function is minimised under a stability constraint [4]. The resulting compensator was an analogue filter with multiple zeros and poles. Nelson and Elliott discussed about compensators based on first-order bilinear or second-order biquad filters [5]. Pawelczyk presented an optimisation method for an analogue controller with one zero and two poles [6]. The optimisation problem was to minimise the sensitivity function at a desired frequency band. The optimisation was constrained by the gain margin and the maximum noise enhancement outside the desired band. For solving the problem, MATLAB Optimisation Toolbox was used. The plant model was obtained by identification. The authors presented an optimisation method for a compensator consisting of several biquad blocks [7]. The cost function to be minimised was a weighted sensitivity function in which the desired frequency band had more weight than the other frequencies. The optimisation constraints were set by the gain and the phase margin. An optimiser for non-smooth, non-convex and inequality constrained problems was used to solve the optimal coefficients and the topology of the biquad blocks. The plant model was a measured frequency response.

In this paper, biquad filters are used as the analogue compensators. An optimisation method based on random search is proposed for optimising the coefficients of a filter. The objective of the optimisation is to minimise the magnitude of the sensitivity function at a desired frequency band. The optimisation is constrained by the maximum amplification of the error. The plant is modelled as a measured frequency response.

## 2 Compensator optimisation

### 2.1 Optimisation problem

In feedback systems, the closed loop transfer function from the disturbance  $D(s)$  to the error  $E(s)$  is given as

$$\frac{E(s)}{D(s)} = \frac{1}{1 + C(s)G(s)}, \quad (1)$$

where  $C(s)$  is the transfer function of the compensator and  $G(s)$  is the transfer function of the plant. The transfer function in Equation (1) is also called the sensitivity function. In the case of maximal rejection of the disturbance, the sensitivity function becomes as small as possible. In feedback systems, however, the sensitivity function can be minimised only at some frequencies while some other frequencies tend to be amplified due to the waterbed effect. [8]

The optimisation objective is to minimise the sum of the squared magnitude of the sensitivity function over a desired frequency band. The cost function can be written as

$$J = \sum_{m=1}^M \left| \frac{1}{1 + C(j\omega_m)G(j\omega_m)} \right|^2, \quad (2)$$

where  $C(j\omega_m)$  and  $G(j\omega_m)$  are the complex frequency response of the compensator and the plant at an angular frequency  $\omega_m$ . The transfer function of the compensator is given as

$$C(s) = K \frac{s^2 + \frac{\omega_z}{Q_z}s + \omega_z^2}{s^2 + \frac{\omega_p}{Q_p}s + \omega_p^2}, \quad (3)$$

where the frequencies of the zero and pole,  $\omega_z$  and  $\omega_p$ , and the corresponding quality factors,  $Q_z$  and  $Q_p$ , are the optimisation variables.

The optimisation is constrained by the maximum amplification of the error over all frequencies. In order to reject the amplification to be less than  $A$ , the Nyquist plot of the open loop system  $C(j\omega)G(j\omega)$  must not pass through a circle having a centre point in  $(-1,0)$  and a radius of  $1/A$  [8]. The constraint can be written as

$$\left| \frac{1}{1 + C(j\omega_n)G(j\omega_n)} \right| < A, n = 1, \dots, N. \quad (4)$$

The sensitivity function cannot thus have values greater than the constraint at any frequency. In the case of violation of the constraint, the gain of the compensator,  $K$ , is decreased so that the constraint is satisfied. With the optimal compensator, the maximum amplification of the error is  $20 \log_{10}(1/A)$  dB.

### 2.2 Optimisation method

For solving the optimisation problem, an optimiser based on random search has been created. Random search methods have previously been applied to optimisation of secondary source and error sensor locations. In random search methods, a large number of solutions are computed and the best solution is taken as the optimum. The best solution found using a random search method is not necessarily the optimal one, but typically it provides performance which is close enough to the optimum for engineering purposes. An example of the random search methods includes the genetic algorithms that are based on biological selection. They work iteratively by forming generations of solutions and evolving the desired properties that optimise the given criterion. [8]

The optimisation method proposed in this paper is related to the genetic algorithms. The method consists of several phases, and in each phase the best solutions found in the previous phase are further refined. In the first phase of the optimisation, a large number of solutions are evaluated in a given parameter space. In the following phases, better solutions are searched in the vicinities of the best solutions in the previous phase. Finally, the solution giving the minimal cost is chosen as the optimum. The optimiser is able to find the global optimum, if the initial parameter space and the number of solutions computed in each phase are sufficiently large. A drawback is that the optimiser requires a large amount of computation.

The performance of the random search based optimiser has been assessed with a Nelder-Mead optimiser used for multidimensional unconstrained nonlinear minimisation. The optimiser is based on direct search method and it is included in the MATLAB as a function called *fminsearch* [9]. The Nelder-Mead optimiser is a local method and may not be able to find the global minimum depending on the initial point. [10]

## 3 Simulations

### 3.1 Test plant

The optimisation method was tested with a test system illustrated in Figure 1. An inertial actuator was installed in a structure, and the aim was to prevent the receiving surface of the structure from vibrating and emitting noise. The inertial actuator consists of two elastic actuator elements and a mass plate located between them. The elastic actuators move the inertial mass plate by becoming thicker or thinner in opposite phases. The error sensor was an accelerometer placed on the receiving surface of the actuator.

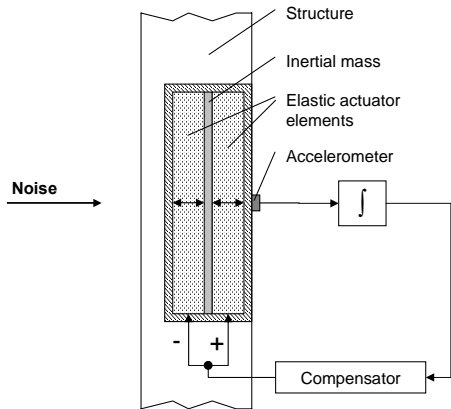


Figure 1: Test system based on inertial actuator and feedback control

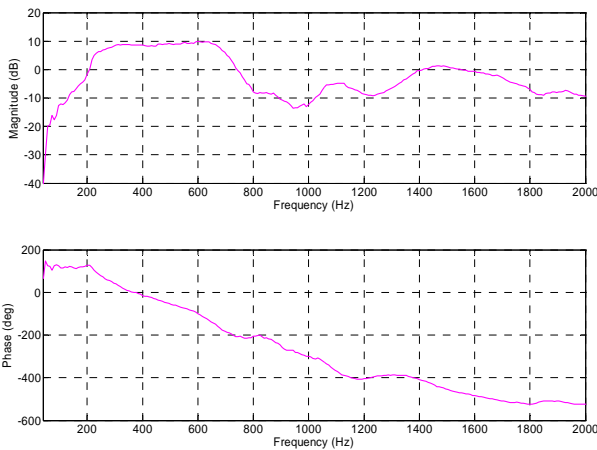


Figure 2: Frequency response of the plant.

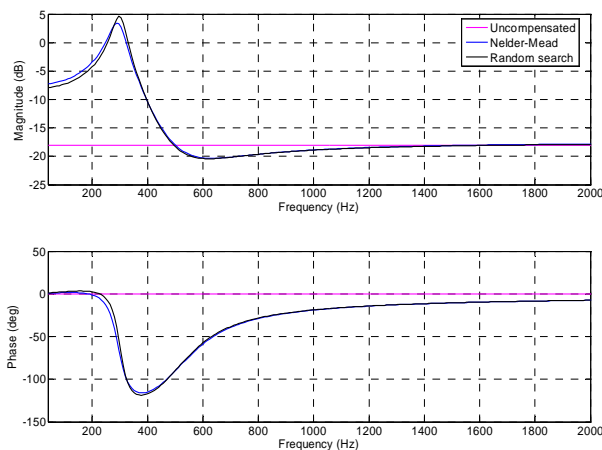


Figure 3: Frequency responses of the compensators.

The velocity feedback control approach was used for driving the actuator. The velocity signal for the compensator was obtained by integrating the accelerometer signal. The compensator for the feedback control was designed using the random search method.

In order to model the plant, the frequency response from the input signal of the actuator to the integrated accelerometer signal was measured. The response of the plant is shown in Figure 2. Between 250 Hz and 700 Hz, the magnitude of the plant is maximal. The frequency band for the maximal attenuation was chosen to be 250-350 Hz. Due to the plant delay, the phase shift steadily increases as a function of frequency, and the optimised frequency band has to be relatively narrow for significant attenuation. The maximum amplification was 3 dB which sets the radius of the circle in the Nyquist plane to be 0.7.

### 3.2 Optimisation results

The optimal compensators were computed using both the random search and the Nelder-Mead optimiser with the given specifications. Using the random search optimiser, the filter coefficients were optimised only once since it was assumed that the optimiser is able to find the global optimum. The optimisation was carried out in three phases with 1000, 300 and 100 evaluations in each. Because the Nelder-Mead optimiser is a local method, it was applied several times with different randomly chosen initial points. The optimisation results including the elapsed time and the final cost are shown in Table 1. The random search method finds a solution giving the minimal cost but the computation of the optimum takes much longer than with the Nelder-Mead method. However, the Nelder-Mead method is highly dependent on the initial point and it has to be applied several times to find the global optimum. Furthermore, the computation time required by the random search method is less than ten seconds which is reasonable in practical applications.

Table 1: Optimisation results

Optimiser	Time (s)	Final cost
Random search	7.6	1.2
Nelder-Mead	1.0	4.3
Nelder-Mead	7.0	1.6
Nelder-Mead	2.4	1.3

The responses of the optimal compensators are given in Figure 3. They are both phase-lag compensators used in feedback active control [4]. The magnitudes of the sensitivity function are shown in Figure 4. They indicate that the random search method performs slightly better and the desired frequency band is attenuated up to 15 dB. Maximum amplification outside the band is 3 dB as specified. With the uncompensated system using only an inverting gain, attenuation of only 3 dB would be achieved with the maximum amplification of 3 dB.

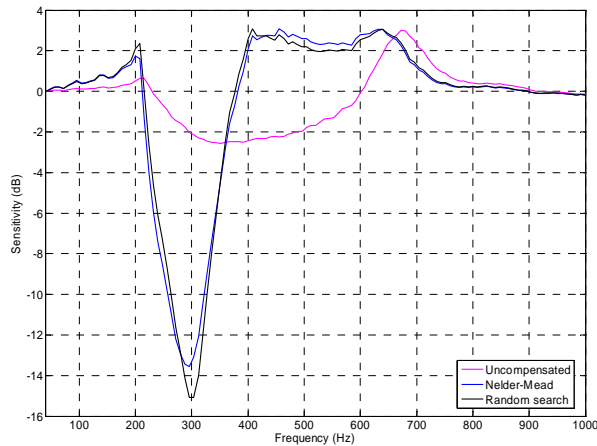


Figure 4: Magnitudes of the sensitivity functions.

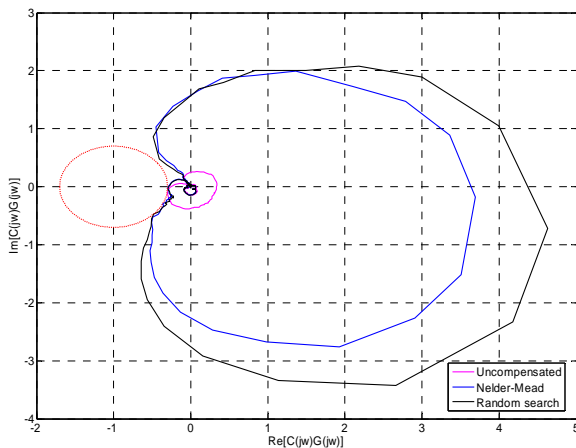


Figure 5: Nyquist plots of the open loop systems.

In Figure 5, the Nyquist plots of the uncompensated and compensated systems are given. They show that the contours of the open loop system do not pass through the circle with a radius of 0.7 corresponding to the desired maximum amplification.

## 4 Summary

In this paper, an optimisation method for biquad filters used as analogue compensators was proposed. The optimisation was based on random search method. The objective was to minimise the sensitivity function at a desired frequency band while a given amplification constraint is satisfied.

The optimiser was evaluated by comparing its performance with a Nelder-Mead optimiser available in MATLAB. Using both optimisers, optimal compensators were designed for a test system consisting of an inertial actuators installed in a structure. The compensator can be implemented using an FPAA which is a digitally adjustable analogue circuit.

The simulation results showed that with the random search method, the desired band is attenuated more effectively than with the Nelder-Mead method. The Nelder-Mead optimiser is a local method and several initial points were required to find the global optimum. The random search method was able to find a better optimum with the cost of long computation time.

## Acknowledgements

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